COMPLEMENTARY THEOREMS OF SURFACE ENWRAPPING EXPRESSED IN GRAPHICAL FORM

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ABSTRACT

In this paper it is presented the issue of surface enwrapping, associated with a pair of rolling centrodes, based on the complementary theorems as "substituting circles family" and "minimum distance" theorem.

The profiling issues are presented for rack gear and gear shaped tools. The solving of these problems is based on the complementary theorems of surface enwrapping, expressed in analytical form.

Using the capabilities of the CATIA design environment are presented specific methods for graphical profiling of rack gear and gear shaped tools.

Examples based on both complementary theorems are presented, establishing the profiles of the tools specifically for the analysed generating types. The examples refer to surfaces composed of line segments and circle arcs. It is emphasized the basic quality of the graphical method – the accuracy of the obtained profile and the intuitive form in which this problem is solved.

KEYWORDS: surface enwrapping, complementary theorems, graphical methods, CATIA

1. COMPLEMENTARY THEOREMS OF SURFACE ENWRAPPING

In this paper, the issues of the two complementary methods are presented. Firstly, the specific theorems for these methods will be presented: the "substituting circles family" theorem and the "minimum distance" theorem [3].

1.1. The theorem of "substituting circles family"

In case of generating profiles associated with a pair of rolling centrodes, this theorem is based on the definition for family of substituting circles.

The family of circles substituting a curve associated with a centrode is the family of circles that, having the centres onto the associated centrode, are, at the same time, tangents to this curve (see figure 1),

Obviously, the circle family is not unique. For practical requirements it is possible to impose a geometrical constraint regarding the position of the

$O_1, ..., O_{i-1}, O_{i+1}$ centres and, in this way, to define the family of substituting circles for the curve C_Σ.
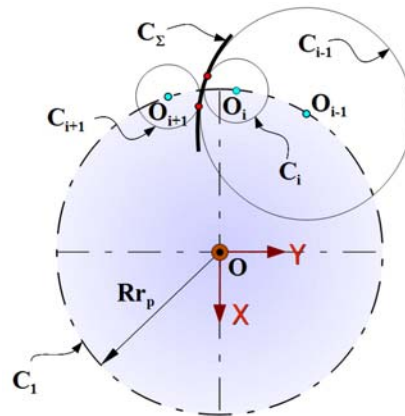

Fig. 1. *Family of substituting circles*

The specific theorem [3, pp. 82-84] is: the enwrapping of a curve associated with a pair of rolling centrodes is the substituting circles family,

transposed, in the rolling motion, onto the conjugated centrode.

For the current circle of the family and of the C_Σ profile, (see figure 2):

$$C_\Sigma \begin{vmatrix} X = X(u); \\ Y = Y(u); \end{vmatrix} \qquad (1)$$

$$C_\Sigma \begin{vmatrix} X = -R_{r_p} \cdot \cos\varphi_1 - r_i \cdot \cos\beta_i; \\ Y = R_{r_p} \cdot \sin\varphi_1 - r_i \cdot \sin\beta_i, \end{vmatrix} \qquad (2)$$

from the tangency constraints, the following are defined:
- the specific enwrapping conditions,

$$\dot{Y}_u \cdot \sin\varphi_1 - \dot{X}_u \cdot \cos\varphi_1 = \\ = \frac{X(u) \cdot \dot{X}_u + Y(u) \cdot \dot{Y}_u}{R_{r_p}}; \qquad (3)$$

- the value of r_i radius,

$$r_i = -\left(R_{r_p} \cdot \cos(\varphi_1 + \beta_i) + \\ +X(u) \cdot \cos\beta_i + Y(u) \cdot \sin\beta_i \right); \qquad (4)$$

- the value of β_i angle,

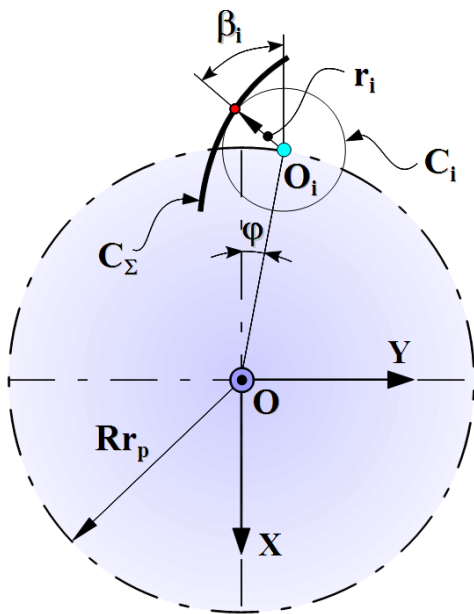$$tg\beta_i = -\frac{\dot{X}_u}{\dot{Y}_u}. \qquad (5)$$

Fig. 2. *Family's circle*

These elements allow to determine the enveloping of C_Σ profile, (1), based on the specific

kinematics for the generation by rolling: generation with rack gear or with gear shaped tools.

1.2. The theorem of "minimum distance"

The specific theorem states that: the enveloping of a profile associated with a pair of rolling centrodes is the locus of the points belonging to the profile for which, in various rolling positions, the distance to the gearing pole is minimum.

Considering a profile C_Σ, see (1) and figure 3, where it is presented a pair of rolling centrodes: C_1 – a circle with radius R_{rp} and C_2 a straight line tangent to C_1 in the gearing pole, joined with the rack gear, it is defined the relative motion:

$$\xi = \omega_3^T(\varphi_1) \cdot \left\| \begin{matrix} X(u) \\ Y(u) \end{matrix} \right\| - \left\| \begin{matrix} -R_{r_p} \\ -R_{r_p} \cdot \varphi_1 \end{matrix} \right\|, \qquad (6)$$

that determines the C_Σ curves' family:

$$(C_\Sigma)_{\varphi_1} \begin{vmatrix} \xi = X(u) \cdot \cos\varphi_1 - \\ -Y(u) \cdot \sin\varphi_1 + R_{r_p}; \\ \eta = X(u) \cdot \sin\varphi_1 + \\ +Y(u) \cdot \cos\varphi_1 + R_{r_p} \cdot \varphi_1. \end{vmatrix} \qquad (7)$$
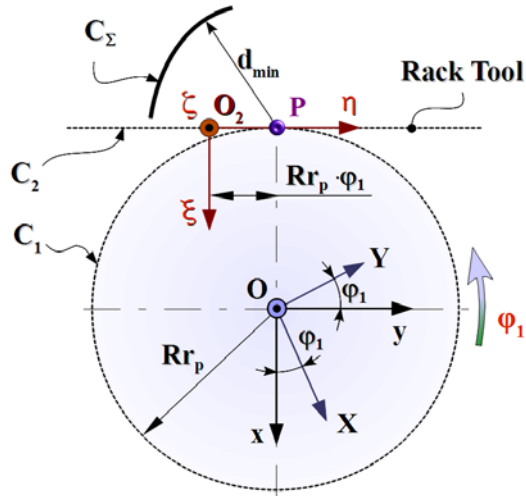
Fig. 3. *"Minimum distance" method; xy – global reference system; XY – mobile reference system, joined with C_Σ; ξη – mobile reference system, joined with the rack gear $(C_\Sigma)_{\varphi1}$*

Let the following be the coordinates of gearing pole:

$$P \begin{vmatrix} \xi_P = 0; \\ \eta_P = -R_{r_p} \cdot \varphi_1, \end{vmatrix} \qquad (8)$$

the distance from the current point onto the C_Σ is given by

$$d = \sqrt{\xi^2(u,\varphi_1) + \left[\eta(u,\varphi_1) - R_{r_p}\cdot\varphi_1\right]} \quad (9)$$

of which minimum value accomplished the condition

$$\xi(u,\varphi_1)\dot{\xi}_u + \left[\eta(u,\varphi_1) - R_{r_p}\varphi_1\right]\dot{\eta}_u = 0, \ (10)$$

representing the specific enwrapping condition.

  Obviously, the distance is measured on the normal draw from the gearing pole to the C_Σ profile.

  The condition (10) may have a specific form at generation with gear shaped tools [3, p. 9].

2. GRAPHICAL METHOD: APLICATIONS, EXAMPLES

Next, profiling examples will be presented for rack gear and gear shaped tools, based on the complementary theorems, solved in the CATIA design environment.

2.1. Rack gear reciprocally enveloping with a hexagonal shaft ("minimum distance" method)

In figure 4, the following are presented:
  - the profile of the hexagonal shaft;
  - *XY* reference system joined with the shaft;
  - C_1 centrode associated with the shaft. This centrode is a circle with radius R_{rp};
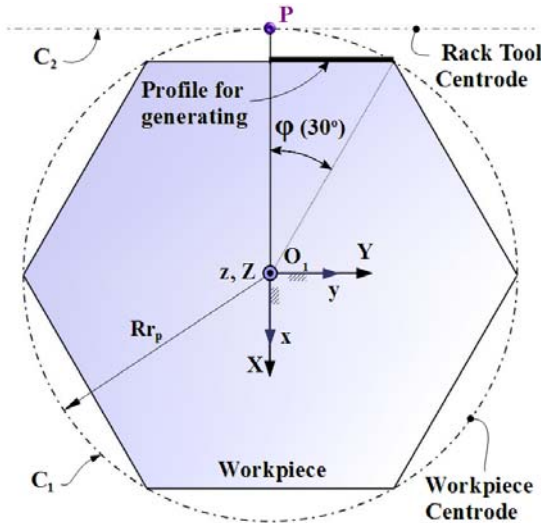 - C_2 centrode associated with the rack tool.

Fig. 4. *The profile of the shaft; C_1 and C_2 centrodes; the gearing pole*

In rolling, for any position, for an increment of the rotation angle $\varphi_i = \varphi/n$ (*n* – integer), are presented positions of the hexagonal shaft side, in *XY* reference system, see figure 5.
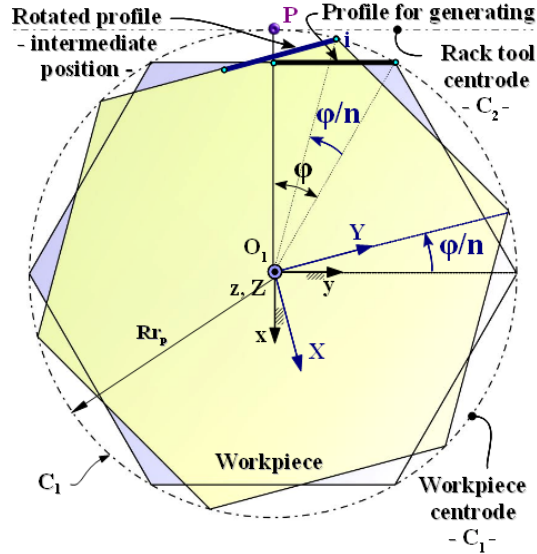

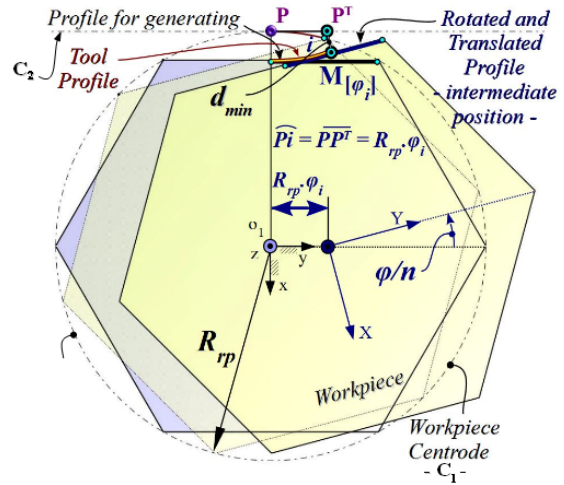**Fig. 5.** *Successive positions in rolling*

Fig. 6. *Normal from profile of the gearing pole; the point onto the tool's profile*

  In figure 7, it is represented a limit position of the hexagonal shaft, for $\varphi = \pi/6$. In this case, the point *M* is onto the rack gear centrode.

  In figure 8, the successive positions of the blank are presented, for a rotation increment of $\varphi_1 = 60/25$, the normals from the relative positions of the gearing pole gearing from the rack centrode to the blank's positions, in rolling.
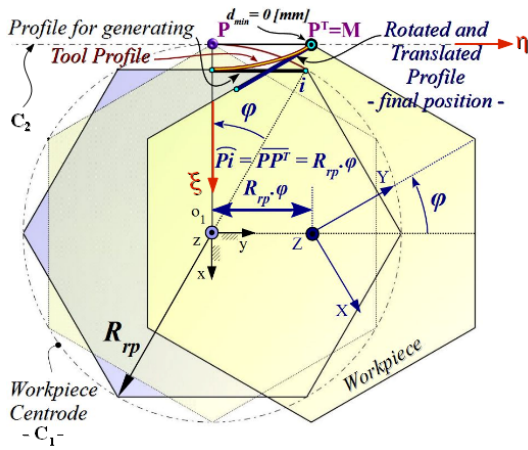
Fig. 7. *The $M(\varphi_i)$ point from the rack's profile, for $\varphi=\pi/6$*

Obviously, the rack profile is obtained by mirror for the opposite segment. The rack gear pitch is $R_{rp} \cdot \dfrac{\pi}{3}$.

In table 1, the coordinates of the rack profile, in the $\xi\eta$ reference system are presented, for the hexagonal shaft with side $R_{rp} = 50$ mm.
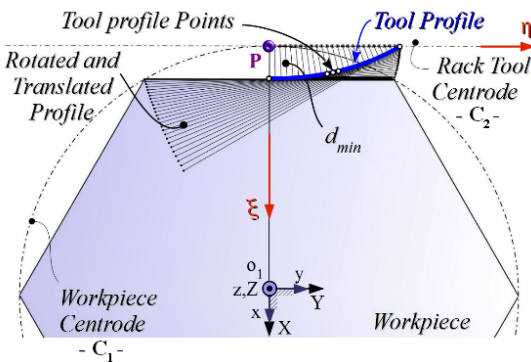

Fig. 8. *The profile of the generating rack gear*

Table 1. *Coordinates of the rack gear's profile*

	η [mm]	ξ [mm]		η [mm]	ξ [mm]
1	0.000	6.699	13	13.798	4.980
2	1.181	6.686	14	14.877	4.692
3	2.361	6.650	15	15.939	4.384
4	3.538	6.588	16	16.984	4.058
5	4.711	6.503	17	18.009	3.714
6	5.879	6.393	18	19.014	3.352
7	7.040	6.260	19	19.998	2.975
8	8.193	6.103	20	20.958	2.584
9	9.338	5.922	21	21.895	2.179
10	10.471	5.720	22	22.806	1.762
11	11.594	5.494	23	23.692	1.333
12	12.703	5.248	24	25.000	0.000

2.2. Gear shaped tool for generating a splined shaft (the "substituting circles family" method)

The splined shafts with parallel flanks have the geometry presented in figure 9. The radii of centrodes C_1 and C_2 are defined.
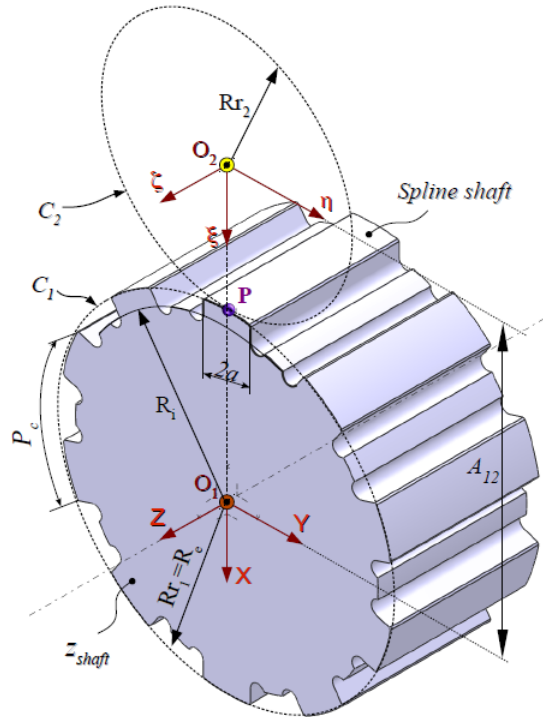

Fig. 9. *Splined shaft with parallel flanks; associated centrodes C_1 and C_2*

Obviously, the R_{r2} radius of the gear shaped tool is determined from the condition that the length of circle $2\pi \cdot R_{r2}$ be equal with an integer number of circular pitch of the teeth - p_C:

$$2\pi \cdot Rr_2 = z_{sc} \cdot P_c \qquad (11)$$

For grinding grooves it is possible to design gaps for entering of the grinding wheel. In the considered example only the issue of the parallel flank of the groove is studied.

The distance between the axes of the two centrodes is

$$A_{12} = R_{r1} + R_{r2}. \qquad (12)$$

The substituting circles have centres onto the C_1 centrode, in points O_1, .., O_i, O_{i+1}. These points have the angle to the centre δ_i, arbitrarily selected, see figure 10.
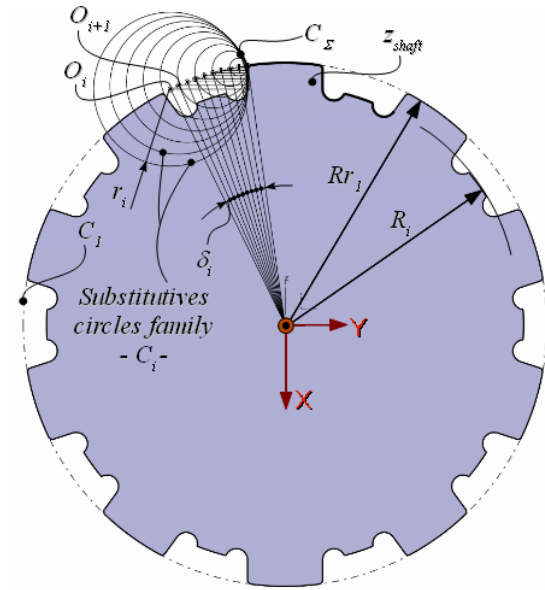
Fig. 10. *Substituting circles family*

In figure 11 there is a detail presenting the tangency points between the family's circles and the profile of the groove flank, C_Σ.

On the Ci circles the tangency points, T_i, T_{i+1}, ..., with the groove flank are determined.
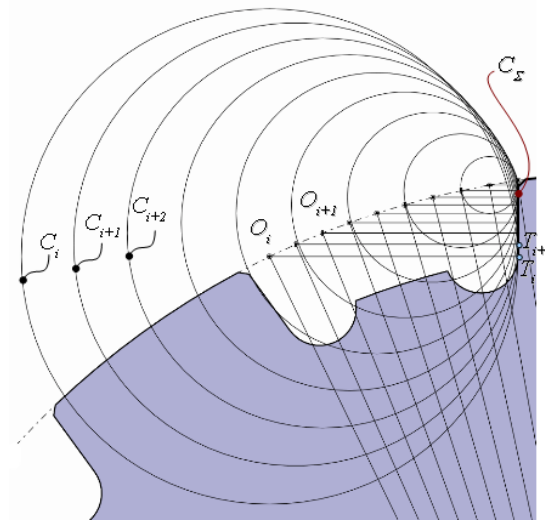

Fig. 11. *Tangency points of the C_i circle with the C_Σ profile*

In figure 12, the circles family is presented transposed by rolling onto the centrode associated with the gear shaped cutting tool, C_2. This centrode is a circle with radius R_{r2}. The assembly of tangency points, T_i, T_{i+1}, ..., after the transposing of the circle's family represents the tooth flank of the gear shape, see figure 13.
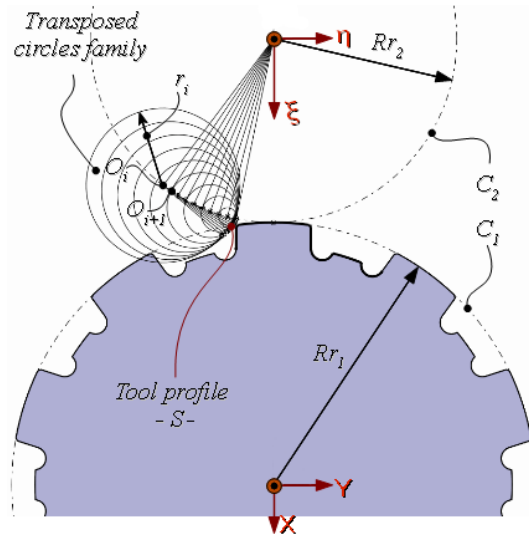

Fig. 12. *Circles family transposed onto C_2; the profile of the gear shaped tool*
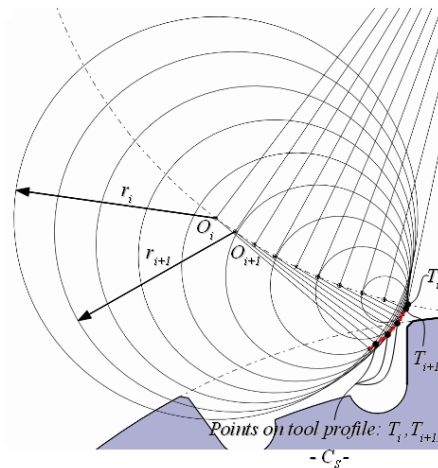

Fig. 13. *Gear shaped tool's profile*

In table 2, the coordinates of the gear shaped tool are presented for a splined shaft with dimensions $R_e = 56$ mm; $R_i = 51$ mm; $a = 16$ mm; $R_{r2} = 39.2$ mm.

Table 2. *Coordinates of the gear shaped tool's profile*

	η [mm]	ξ [mm]
1	-7,9710	38,3810
2	-8,1840	38,8730
3	-8,4430	39,3430
4	-8,7400	39,7890
5	-9,0700	40,2120
6	-9,4270	40,6120
7	-9,8080	40,9900
8	-10,2100	41,3450
9	-10,6310	41,6770
10	-11,0680	41,9880

3. CONCLUSIONS

The complementary methods entitled "substituting circles family" and "minimum distance" allow one to approach a variety of issues regarding profiles (or surfaces) joined with a pair of rolling centrodes.

Profiling examples were presented for rack gear and gear shaped tools for generating a whirl of external surfaces.

The graphical methods developed in CATIA allow obtaining rigorous results regarding the form and dimensions of the profile generated by enwrapping.

The graphical method is very intuitive, avoiding the profiling errors for these tools.

The graphical method developed in CATIA is characterised by the fact that it is very precise (the geometrical features are determined with a precision of 10^{-12} mm, which exceeds the determining precision of surfaces in the generating process by numerical methods).

More, the graphical methods are easy to apply, because they are based on the capabilities of the graphical design environment.

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